

Times of Flight between a Source and a Detector observed from a GPS satellite.

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(Dated: 12 October 2011)

The Michelson-Morley experiment shows that the experimental outcome of an interference experiment does not depend on the constant velocity of the setup with respect to an inertial frame of reference. From this one can conclude the existence of an invariant velocity of light. However it does not follow from their experiment that a time-of-flight is reference frame independent. In fact the theory of special relativity predicts that the distance between the production location of a particle and the detection location will be changed in all reference frames which have a velocity component parallel to the baseline separating source and detector in a foton time-of-flight experiment. For the OPERA experiment we find that the associated correction is in the order of 32 ns. Because, judging from the information provided, the correction needs to be applied twice in the OPERA experiment the total correction to the final results is in the order of 64 ns. Thus bringing the apparent velocities of neutrino's back to a value not significantly different from the speed of light. We end this short letter by suggesting an analysis of the experimental data which would illustrate the effects described.

Keywords: special relativity, interpretation Michelson-Morley experiment, OPERA, neutrino velocity

The Michelson and Morley [4] experiment demonstrated the speed of light is the same in all inertial frame of reference and on this axiom Einstein built special relativity [1]. Although the velocity of light is invariant under such a change of reference frame, special relativity does not preserve distance and time separately. In fact to make the outcome of the interference pattern in the Michelson-Morley experiment reference frame independent these coordinates are subject to joint Lorentz transformations rendering the speed of light invariant. In addition the description of the events taking place differs between different frames-of-reference, in other words a change in perspective changes the scenario. This scenario change becomes important if we want to calculate the speed of particles using a source A and a detector B separated by a fixed distance $S_{baseline}$ in their baseline reference frame and relying on a clock which is also moving but with a velocity

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v from source to detector. In the reference frame of the clock the source and detector positions are changing, and from the perspective of the clock the detector is moving towards the source and consequently the distance traveled by the particles as observed from the clock is shorter than the distance separating the source and detector in the clock reference frame. We calculate the time-of-flight in the moving clock reference frame for fotons, i.e. particles moving at the speed of light from A to B, and compare it the time-of-flight estimate for fotons in the baseline reference frame.

The distance S_{clock} between the source and detector in the clock reference frame is related by Lorentz contraction to the distance $S_{baseline}$ in the baseline reference frame [2]:

$$S_{clock} = \gamma S_{baseline} \quad (1)$$

with $\gamma = \sqrt{1 - v^2/c^2}$. From the perspective of the clock the detector at B moves towards location A at a speed v . And we find that the foton will reach the detector when the sum of the distances covered by the detector and the foton equals the original separation, i.e

$$\tau_{clock}c + \tau_{clock}v = S_{clock} \quad (2)$$

with τ_{clock} the time of flight in the clock reference frame. From this we find

$$\tau_{clock} = \frac{S_{clock}}{c + v} = S_{baseline} \frac{\gamma}{c + v}. \quad (3)$$

The authors of the OPERA paper [5] seem to include a correction for the Lorentz transformations, but they not correct for the change in scenario. And because they project back the time of provided by the moving clock to the baseline they seem to incorrectly assume that the outcome of their experiment should be equivalent to that using a clock in the baseline reference system,

$$\tau_{baseline} = \frac{S_{baseline}}{c}, \quad (4)$$

but in fact they should observe the Lorentz transformation corrected time-of-flight as measured by the moving clock, i.e.

$$\tau_{observed} = \frac{\tau_{clock}}{\gamma} = \frac{S_{baseline}}{c + v}. \quad (5)$$

The difference between the observed time-of-flight $\tau_{observed}$ and the baseline time-of-flight $\tau_{baseline}$ is given by,

$$\epsilon = \frac{S_{baseline}}{c} \left(1 - \frac{c}{c + v} \right). \quad (6)$$

To verify that this explains the observed error we need to calculate the quantity ϵ and we need to identify where and how often this error is made and if there are no cancellations between the different errors. We start with the calculation of ϵ . The clocks in the OPERA experiment are orbiting the earth in GPS satellites. The orbits of these satellites are at $20.2 \cdot 10^6$ m from the earth's surface in a fixed planes inclined 55° from the equator with an orbital period of 11 h 58 min [3]. This implies that they fly predominantly West to East when they are in view of CERN and Gran Sasso, which is roughly parallel to the line CERN-Gran Sasso. The radius of a GPS satellite's orbit is found by adding to the height the radius of the earth $6.4 \cdot 10^6$ m which yields a radius of $26.6 \cdot 10^6$ m. The velocity of the GPS satellites v is therefore approximately $v = 2\pi R/T = 2\pi \cdot 26.6 \cdot 10^6 \text{ m}/(12 \cdot 60 \cdot 60 \text{ s}) \approx 3.9 \cdot 10^3 \text{ m/s}$, further using $S_{baseline} = 7.3 \cdot 10^5 \text{ m}$ and the speed-of-light $c = 3.0 \cdot 10^8 \text{ m/s}$ we obtain:

$$\epsilon = 32 \text{ ns.} \tag{7}$$

In other words the observed time of flight should be about 32 ns shorter than the time-of-flight using a baseline bound clock. Now we should examine the experiment again to identify potential other locations where these types of error can be made. Most of the corrections to the result are estimated using baseline based clocks, these corrections do not change the expected observed time-of-flight. However to relate baseline time to GPS clock time the GPS clock time is corrected for time-of-flight of the radio signals. It is likely that this is also done using the baseline reference frame where the clock reference frame should be used. As this involves the same clock and the same events the error should be the same, i.e. from the baseline reference frame the time-of-flight of the radio signals in the clock reference frame is overestimated by ϵ and hence we expect that the total error is in the order of $2\epsilon = 64 \text{ ns}$.

I. CONCLUDING REMARKS

We showed that in the OPERA experiment the baseline time-of-flight is incorrectly identified with the Lorentz transformation corrected time-of-flight as measured from a clock in a non-stationary orbit and in fact exceeds it by at maximum 64 ns. The calculation presented contain some simplifying assumptions, a full treatment should take into account the varying angle between the GPS satellite's velocity vector and the CERN-Gran Sasso baseline. We expect that such a full treatment will find somewhat lower value for the average correction. This is because the velocity of the GPS satellite is most of the time not fully aligned with the CERN-Gran Sasso baseline. In

addition full analysis should be able to predict the correlation between the GPS satellite position(s) and the observed time-of-flight.

We know from special relativity that time is reference frame specific. This paper shows that Coordinated Universal Time (UTC) happens to be less universal than the name suggests, and that we have to take in to account where our clocks are located. Finally, making all calculations from the correct reference frame might also lead to further improvement of the accuracy of GPS systems as the errors reported here for the time-of-flight amount to a ± 18 m difference in location.

Acknowledgments

The author is a member of the Sensor City (Assen, The Netherlands) project team which is funded by the European Union, European Regional Development Fund, the Dutch Ministry for Economical Affairs and the Northern Netherlands Provinces. The author likes to thank Tjeerd Andringa for discussions during the early phase of this research.

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- [1] A Einstein. Zur Elektrodynamik bewegter Körper. *Annalen der Physik*, 17:891, 1905.
 - [2] C Kittel, W Knight, M.A. Ruderman, A.C. Helmholz, and B.J. Moyer. *Mechanics, Berkeley Physics Course, Volume 1 2nd ed.* McGraw-Hill, Inc., 1973.
 - [3] M.A. Lombardi, L.M. Nelson, A.N. Novick, and V.S. Zhang. Time and frequency measurements using the global positioning system. *The International Journal Of Metrology*, pages 26–33, 2001.
 - [4] AA Michelson and EW Morley. On the relative motion of the earth and luminiferous ether. *American Journal of Science*, 203:333–345, 1887.
 - [5] The OPERA Collaboration. Measurement of the neutrino velocity with the OPERA detector in the CNGS beam. *arXiv:1109.4897v1 [hep-ex]*, 2011.