After the nth iteration, we get

$$Z_{n+1} = Z_1(F'(\zeta_{20}))^n. \tag{23}$$

For the case of small losses, ($g_1 \gg \epsilon_1$, $g_2 \ll \epsilon_2$), we have

$$F'(\zeta_{20}) \approx -1 + \frac{\beta_0 D\epsilon_2}{2\sqrt{2g_1}}(1-j) + O\left(\frac{1}{g^{3/2}}\right). \tag{24}$$

$|F'|$ is less than unity, and the process will converge. However, F' will be close to -1, and the consecutive values of ζ_2 will oscillate around the correct value ζ_{20}. By modifying the procedure so that after the two first steps the computer calculates

$$Z_4 = \frac{Z_2 + Z_3}{2} = \frac{F'(1+F')}{2} Z_1 \tag{25}$$

and then follows the same cycle, we can apply the same reasoning on $(1+F')F'/2$ as previously on F'.

For small loses, we get

$$\frac{F'(1+F')}{2} \approx \frac{\beta_0 D\epsilon_2}{2\sqrt{2g_1}}(j-1) + O\left(\frac{1}{g_1}\right). \tag{26}$$

It is obvious that this modification will bring about a faster convergence, at least for small losses, and it turned out to be adequate for all values on the parameters used in the computations.

Acknowledgment

The author is grateful for the assistance given by the computer group at TRG under the direction of J. Nihen, and for many helpful suggestions given by F. J. Zucker of AFCRL and Dr. A. F. Kay of TRG.

Very-Low-Frequency Propagation Below the Bottom of the Sea*

H. MOTT†, SENIOR MEMBER, IEEE, AND A. W. BIGGS‡, MEMBER, IEEE

Summary—Radio-wave propagation at very low frequencies (VLF) in the stratified rock below the bottom of the sea is studied. A reasonable assumption of extremely low electrical conductivity in the stratified rock is based upon available geological data. The surface wave traveling along the interface between this region of low conductivity and the highly conducting sea is compared with the vertically polarized ground wave found in VLF radio-wave propagation at the surface of the earth. When extremely low frequencies (ELF) are transmitted, the highly conducting layer found at greater depths below the bottom of the sea forms the lower surface of a spherical waveguide. This waveguide at ELF supports a propagation mode similar to the mode existing at VLF between the surface of the earth and the lower boundary of the ionosphere. The similarity in propagation mechanisms leads to the name "inverted ionosphere" (described by Wheeler [1]) for the underground region.

The sea or relatively highly conducting soil at the surface of the earth is an almost impregnable shield against atmospheric noise and effects from sudden ionospheric disturbances or solar flares. In addition to providing a noise-free medium, the sea has the advantage that construction costs are much less than those of a VLF transmitter at the earth's surface.

Presumably communication between shore installations and submarines on the floor of the ocean could be achieved with the inverse ionosphere. The power requirement for such communication with existing VLF transmitters at the earth's surface renders such transmission unattainable.

* Received November 15, 1962; revised manuscript received January 12, 1963.

† Electrical Engineering Department, University of Alabama, Tuscaloosa; formerly of the Aero-Space Division, Boeing Company, Seattle, Wash.

‡ Aero-Space Division, The Boeing Company, Seattle, Wash.

Introduction

SOME EVIDENCE exists that the electrical conductivity of the earth's deep crust and upper mantle is low enough to make feasible underground radio-wave communication over distances of 1000–2000 km, both under the sea and the continents. This possibility was suggested by Wheeler [1], who has called the lower boundary of the propagation region an "inverted ionosphere" or "thermal ionosphere." This terminology is somewhat misleading because the resistivity and dielectric constant relationships differ considerably from those of the ionosphere; but a similar, though not directly analogous, propagation effect occurs. It has the advantages of offering secure communications, providing a communication system if above-ground systems are disrupted, and potentially offering much lower noise levels than above-ground systems.

The upper boundary of the propagation region is the wet, highly conducting region of the earth's continental surface or ocean bottom. The propagation region is composed of dry, low-temperature granitic or basaltic rocks extending to a depth of 20–70 km. The lower boundary is formed by ultrabasic rocks at higher temperatures, thus having greater electrical conductivity.

Propagation Analysis

An appropriate arrangement of short monopole antennas is indicated in Fig. 1. The antennas should extend about 1 or 2 km into the low conductivity region, whose upper boundary lies about 1 km below the ocean floor and about 0–5 km below the continental surface.

The most important earth property affecting this type of propagation is its electrical conductivity, particularly that of the region of low conductivity. Waves passing through a conducting medium have part of their energy transformed into heat. This energy absorption is greater for higher conductivity values and sets an upper limit of about 10^{-6} mhos/m for the low conductivity region if long-distance communication is to be obtained. It is unfortunate that our knowledge of earth conductivity is not extensive. A brief sketch of the earth's near-surface structure and the more important attempts to measure and estimate its conductivity is given here.

Under continents the top earth layer is sedimentary with a thickness 0–5 km. Next is a granitic layer about 15–21 km thick. The third layer is a basaltic layer (tachylyte or diorite) 11–19 km thick. The last layer of importance to this discussion is dunite, possibly with eclogite (both ultra-basic) near the top, extending to about 2900 km. Under oceans, the sedimentary layer is thin, the granitic layer may be absent, and the tachylyte (basalt in vitreous condition) replaced by a crystalline basalt [2]. The ocean sediments have a depth of about 1 km [3]. The seismic interface at about 15 km (ocean) or 35 km (continent), marking the upper boundary of the dunite layer extending to 2900 km, is the Mohorovicic discontinuity. The dunite layer is named the mantle, and all above the Mohorovicic discontinuity is the earth's crust.

The earth's sediments are relatively porous and easily penetrated by water. Sediment conductivity is quite high and easily measured.

Direct measurements of deeper earth conductivities by drilling would be ideal from the standpoint of accuracy. Oil drilling, unfortunately, is normally confined to sedimentary layers having relatively high conductivity. In addition, the deepest wells are less than 7 km deep [4]. Thus, oil-well electrical logs have limited application to our problem.

Estimates of conductivity based on indirect methods are shown in Fig. 2. The line marked 1 indicates measured earth-surface conductivities ranging from 10^{-8} for poorly conducting earth to 4 mhos/m for sea water.

Curve 2 is from measurements of Hughes [5], [6] on

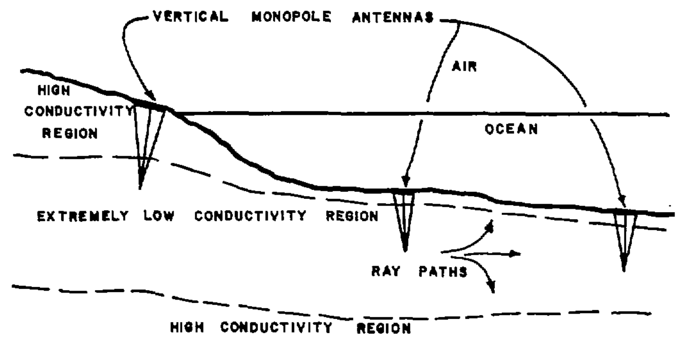

Fig. 1—Upper-earth section showing communication region.

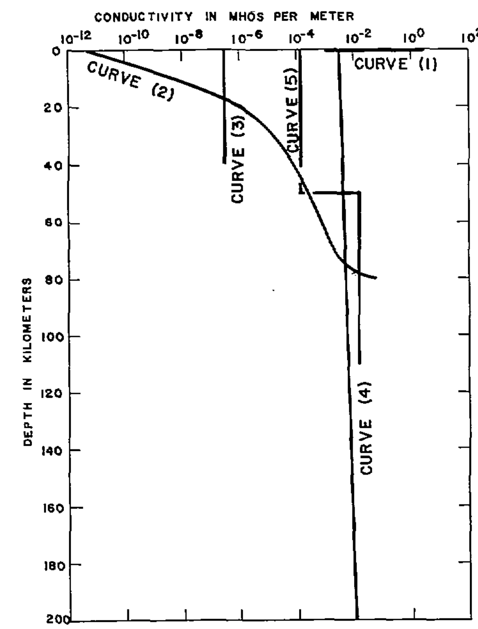

Fig. 2—Earth conductivity estimates.

the conductivity of olivines as a function of temperature in the range 0–1500°C. Hughes found similar results for other ferro-magnesium materials. The assumption is made that such materials are an important constituent of the earth's deeper crust and mantle. In drawing Curve 2 a temperature distribution must be assumed. MacDonald's work [6] on temperatures is quite extensive. He uses estimates of the distribution of radioactive sources (based on earth's-surface distributions), measured thermal conductivities, etc., to arrive at temperature distributions. He estimates the temperature at 30 km to be of the order to 500°C. Near the surface, temperature increases with depth in an approximately linear way, but at perhaps 100 km it begins to fall below the value predicted by a linear increase. Curve 2 is based on a temperature of 0°C at the ocean bottom, 500°C at 26 km below the bottom, and a linear temperature change with depth. It is discontinued at 80 km because of doubt about the usefulness of the linear temperature assumption. At greater depths the fall-off of temperature below the linear values predicted by MacDonald would make Curve 2 turn and more nearly parallel Curve 4.

Garland and Webster [7], [8], using a method often attributed to Cagniard [9], estimated Curve 3 as a maximum conductivity of the Pre-Cambrian basement rocks of west central Canada. Cagniard's method involves the comparison of the horizontal surface components of the magnetic and electric fields associated with the flow of natural subsurface earth currents. Some of the limitations of this method, and a thorough review of the present state of knowledge of magneto-telluric fields, are treated by Wait in a recent paper [10]. For a homogeneous earth the relation is

$$\frac{|E|}{|H|} = \sqrt{\frac{2\mu}{\sigma T}}$$

where E and H are mutually perpendicular surface fields, μ is the earth's magnetic permeability, T is the period of the harmonic current being measured, and σ is the conductivity. For a layered structure such as the earth, the theoretical expressions are more complicated, and in use it is necessary to fit data to curves drawn for assumed earth models. Garland and Webster assume a 2-layered structure with high conductivity (measured surface values) down to a depth of 6000–8000 ft (top surface of basement rock, measured seismically) and a low-conductivity region extending to infinity. Their measurement indicated that this region's conductivity is at most 3×10^{-7} mhos/m. They suggest that conductivity increases below 40 km [8].

Cantwell and Madden [11], using the same method but a different earth model, arrived at substantially different conductivities, shown as Curve 5 of Fig. 2. They also assumed a 2-layer model with the upper layer, of depth 70 km, having the lower conductivity (the reverse of the assumption of Garland and Webster) and with the lower layer extending to infinity. They arrived at resistivity values of 8000 ohm meters for the upper layer and less than 80 ohm meters[1] for the lower. Wait [10] has shown, however, that a 3-layered model with a region of low conductivity sandwiched between two regions of higher conductivity (such as is assumed in this paper) is, for sufficiently low conductivity of the intermediate region, equivalent to a 2-layered model with parameters σ_1, σ_3, and h_1. (Subscript 1 refers to conductivity and thickness of the uppermost layer. The subscript 3 refers to the bottom layer.) The resulting curves are then independent of σ_2 and h_2. Wait's criterion is that

$$(\sigma_2 \mu_0 \omega)^{1/2} h_2 \ll 1,$$

which is satisfied for a conductivity of 10^{-7} mhos/m, a frequency of 1 cps, and a thickness of 10 km. In the light of Wait's discussion it seems clear that the existence of a low-conductivity region underlying the more highly-conducting upper earth layer could have been overlooked by Cantwell and Madden.

In this paper Wait [10] also gives the quantitative findings of other investigators, notably Scholte and Veldkamp, who arrived at an upper-layer conductivity of 2 mhos/m (in a 600 m thickness) and a lower-layer conductivity of 10^{-1} mhos/m; and Vladimirov, who found an upper-layer value of 5×10^{-2} mhos/m in a thickness of 450 m and a lower-layer value of approximately zero.

Lahiri and Price [6], [12] give Curve 4 with values corroborated by MacDonald [6]. They use the daily and magnetic storm variations of the earth's magnetic field, separating the exciting components and those caused by earth currents induced by the exciting terms. Lahiri and Price solve the problem of fields induced in a spherically symmetric conductor and fit measured field data to the solution to estimate conductivity from 200–800 km. MacDonald extends this work to 2800 km and estimates conductivity at that depth of 600–700 mhos/m. The extension from 200 km to 0 km merely fits an empirical curve based on values at greater depths.

Near the ocean bottom the data of Hughes (Curve 2) is invalid because of water penetrating the sediments and upper rocks. Conductivity in that region can then be expected to be greater than that predicted by Curve 2, but almost certainly less than near-surface values. Values near the Garland and Webster measurement will be adopted for this paper. At depths of about 10–60 km, Curve 2 is probably the best estimate available. At still greater depths conductivity should be less than that predicted by a continuation of Curve 2, and Curve 4 becomes the better estimate. These considerations lead to the assumed conductivity model of Fig. 3. The characteristics of wave propagation will depend strongly on frequency, assuming the conductivity model of Fig. 3. At a frequency of 100 kc the wavelength inside the earth is about 1.2 km, assuming a relative dielectric constant of 6 (typical of dry, poorly conducting ground at the earth's surface). This is small compared to the depth of the region of low conductivity. One might expect then that the increase in conductivity below 10–20 km will not have a great effect on the propagation properties. The increase in conductivity will increase the index of refraction with increasing depth; consequently, rays from the transmitting antenna directed below the horizontal and entering the region of higher conductivity will be refracted toward the earth's center and absorbed with little or no reflections toward the receiver. The physical model assumed, therefore, for propagation of frequencies of order of 100 kc is a plane half space of conductivity $10^{-6}-10^{-7}$ mhos/m bounded at the top by a layer having conductivity 4 mhos/m. A plane surface can be used instead of the more complex spherical surface for propagation distances not exceeding one quarter of the earth's circumference [13]. The assump-

[1] Wait [10] reports that Fig. 3 of Cantwell and Madden's paper is apparently mislabeled and that this should be 8 ohm meters.

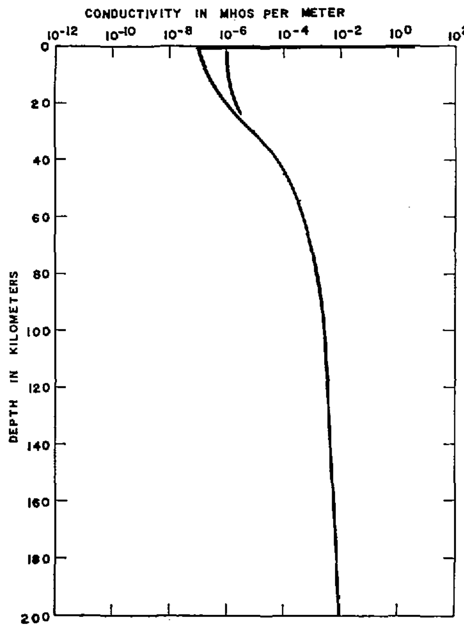

Fig. 3—Electrical conductivity below the sea bottom.

TABLE I

Absorption Loss per KM Path Length

Conductivity, mhos/m	Frequency, cps	Loss, db/km
10^{-8}	10^2	6.6×10^{-3}
10^{-8}	10^3	6.7×10^{-3}
10^{-8}	10^4	6.7×10^{-3}
10^{-8}	10^5	6.7×10^{-3}
10^{-7}	10^2	4.6×10^{-2}
10^{-7}	10^3	6.6×10^{-2}
10^{-7}	10^4	6.7×10^{-2}
10^{-7}	10^5	6.7×10^{-2}
10^{-6}	10^2	0.17
10^{-6}	10^3	0.46
10^{-6}	10^4	0.66
10^{-6}	10^5	0.67
10^{-5}	10^2	0.55
10^{-5}	10^3	1.7
10^{-5}	10^4	4.6
10^{-5}	10^5	6.6

tion is next made that the low conductivity of the half space attenuates the wave, but does not affect significantly the propagation mechanism. The ultimate model is that of a plane lossless (absorption losses to be added later) half space bounded by a surface of finite conductivity. Propagation in such a region is by a surface wave guided along the interface. This type of propagation is utilized by commercial radio transmitters in the ground wave mode. Numerical solutions to the problem are given by Norton [14]. For frequencies up to 100 kc the inverse distance-squared loss is the only significant factor in determining signal strength. The additional loss due to the finite conductivity of the half space is shown for various frequencies and conductivities in Table I. Clearly, the conductivity of the region of lowest conductivity is of critical importance for this type of propagation. Conductivities significantly above 10^{-6} mhos/m will not allow long-distance communication.

At lower frequencies a wavelength becomes long in relation to the thickness of the low-conductivity region (wavelength is 12 km at 10 kc for relative dielectric constant of 6 and conductivity of 10^{-6} mhos/m or less). In addition, the earth's conductivity and refractive index change significantly with depth in a distance of 1 wavelength. Thus a lower boundary somewhere in the 20–70 km region is added to the preceding problem. Analysis is quite difficult but a fairly good approximation can be obtained by assuming as a model a lossless region bounded by two infinite conducting plane surfaces. A suitable treatment of this model was suggested by Budden [15], who erroneously included negative order modes in his solution. Budden [16] later modified his treatment by correctly deforming the integration contour to exclude the negative order poles. A more complete treatment was given by Wait [17] for the modal solution of the above problem. An excellent summary and up-to-date picture of the VLF model was recently discussed by Wait [18] for the IRE Wave Propagation Committee. In the undersea propagation region it appears that a transverse electromagnetic (TEM) waveguide mode will be important [1]. This is characterized by a vertical electric field and an azimuthal magnetic field with amplitudes dependent only on radial distance from the transmitting antenna. Propagation is radially outward from the transmitter. The power per unit area at any point falls off inversely with distance, rather than inversely with the square of distance as in the first case. This offers a gain increase of 10–15 db, depending on the waveguide height, over a wave of the same frequency guided by one surface for a distance of 1000 km. The results will be modified by absorption in the medium between the two boundaries (Table I). This absorption loss is much less for the low frequencies treated by the waveguide theory than for the higher frequencies which may be guided by one surface only. Results must also be modified by wall losses in the imperfectly conducting boundaries. The assumption is made here that such losses can be estimated by Norton's work for surface waves. Wall losses will then be negligible at 1 kc and perhaps 10–20 kc for a distance of 1000 km.

Lower absorption attenuation (Table I) and a ducted propagation gain of 10–15 db favors the use of lower frequencies if earth conductivity is high enough to cause troublesome attenuation of the higher frequencies over the desired signal distance. Factors favoring the higher frequencies are greater signal bandwidth and higher antenna efficiencies for the same physical antenna length.

It is worth pointing out that propagation is not dependent on the existence of the lower conducting boundary. The only requirement is for a region of low conductivity bounded by the highly conducting upper surface.

A major problem affecting above-earth communication is radio noise, both external to the receiver and internal. The energy of the desired signal at the receiver should be greater than the noise energy for intelligible signal reception. External noise has three sources: atmospheric (thunderstorms), cosmic and man-made. By choosing suitable nonmetropolitan receiving sites, man-made noise can normally be kept below the noise from other sources [19]. At such locations, thunderstorms (at distances up to thousands of kilometers) normally are the principal sources of noise below 15 Mc. For frequencies between 15 Mc and 150 Mc the noise is chiefly from cosmic sources [19]. Receiver noise, with good system design, is due to thermally-caused random motion of electrons in resistors, and to various fluctuation noises in tubes [19].

An additional external noise is the thermal noise associated with the antenna radiation resistance. This noise power originates in the antenna surroundings which are capable of radiating power to the antenna. The available noise power is kTB, where k is Boltzmann's constant, T is an average temperature of the antenna surroundings (often taken as 288°K), and B is the receiver bandwidth in cps [20]. The value of this available noise power for a 1000 cps bandwidth is 3.95×10^{-18} w. Atmospheric and internal receiver noises are commonly described by giving their power at some frequency as a ratio to the thermal radiation resistance noise power. The internally-generated receiver noise is referred back to the antenna terminals before the comparison is made. The available noise power from all sources is assumed to be proportional to the receiver bandwidth (a characteristic of random noise), as is the radiation resistance power reference. Then the ratios describing atmospheric and receiver noise are independent of receiver bandwidth [21]. Fig. 4 shows expected values of atmospheric radio noise (in decibels above antenna thermal noise) as a function of frequency for the local time 0000–0400 and 2000–2400 (all seasons) and 0400–0800 and 1600–2000 (autumn and winter).

In the standard method of noise prediction, contours of equal noise grade are drawn throughout the world and the appropriate noise grade curve of Fig. 4 is assumed to apply. For example, a contour of noise grade 70 lies almost completely in the United States from 0000–0400 and from 2000–2400 in December, January and February. Noise grade contours and data in the 10 kc$-$1 Mc range were taken from a report of the C.C.I.R. (International Radio Consultative Committee) [21] and are the latest data available. Actual noise measurements have been made only for a few places in the world, as might be expected, and the noise predictions for many parts of the world are interpolated from a knowledge of thunderstorm distribution and radio propagation characteristics [21]. In the 1- to 10-kc range noise information is rare. The C.C.I.R. curves were extended down to 1 kc by estimates based on the measurements of Watt and Maxwell [22], [23]. They, to a large extent, measured peak-noise electric field strengths instead of noise power, but on the basis of some noise power measurements they predict a noise behavior with frequency similar to their peak field strength measurements. Their measurements were made at Boulder, Colo. (approximate noise grade 70), and the assumption is made that the results can be extended to other noise grades.

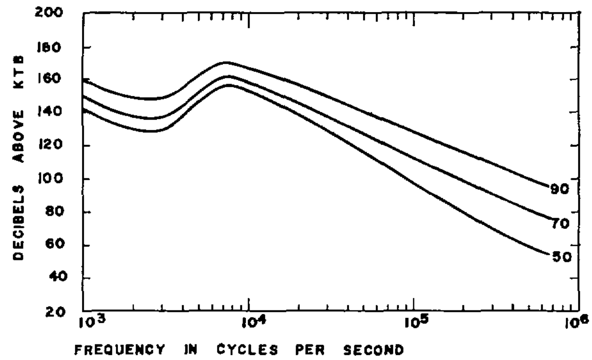

Fig. 4—Expected atmospheric noise.

Of greatest interest to this report is the realization that atmospheric noise power may be as great as 200 db (Fig. 4). In marked contrast, radio receiver noise figures range from 8–30 db [19]. It will be shown that atmospheric noise is severely attenuated in passing into the underground propagation region considered here. If no comparable radio noise is generated inside the earth (at the frequencies of interest), underground propagation will have a very great advantage over above-ground systems in this respect.

Horizontally-polarized noise components (electric field vector horizontal) cannot propagate to great distances over the earth's surface [24]. Thus the atmospheric noise at any place is substantially vertically polarized with only a relatively small horizontal component necessary to meet the boundary conditions at the earth's surface. Only a horizontally polarized wave can be propagated downward through the earth, however. The ratio of power contained in the wave above earth to that of the vertically-downward propagating wave below the earth is shown by Table II. On its downward path to the underground propagation region the noise is absorptively attenuated by amounts shown in Table III (plane wave calculations). These are the only noise losses for a submarine electromagnetically coupled to the underground propagation region, by a loop antenna, for example, as suggested by Wheeler [1]. Such coupling should then be avoided by submarines at shallow depths.

The maximum attenuation of the noise power as it passes from the sea or upper-earth conduction region into the underground propagation region is infinite if a uniform plane wave traveling vertically downward is assumed, together with a plane, horizontal interface and

TABLE II

NOISE POWER LOSS, AIR-TO-EARTH TRANSITION

Sea Water		Earth, Conductivity $=10^{-2}$	
f	db loss	f	db loss
10^2	44.3	10^2	31.3
10^3	39.3	10^3	26.3
10^4	34.3	10^4	21.3
10^5	29.3	10^5	16.3

TABLE III

NOISE POWER ATTENUATION BY ABSORPTION

Sea Water		Earth, Conductivity $=10^{-2}$	
f	db/m loss	f	loss, db/m
10^2	0.345	10^2	0.0171
10^3	1.10	10^3	0.0545
10^4	3.45	10^4	0.171
10^5	11.0	10^5	0.545

TABLE IV

MINIMUM NOISE POWER LOSS, CONDUCTING-TO-PROPAGATION-REGION TRANSITION

	Sea Water		Earth, Conductivity $=10^{-2}$	
Propagation Region Conductivity	Frequency	db loss	Frequency	db loss
10^{-7}	10^2	31.9	10^2	18.9
10^{-7}	10^3	29.3	10^3	16.3
10^{-7}	10^4	24.4	10^4	11.4
10^{-7}	10^5	19.4	10^5	6.4
10^{-6}	10^2	27.0	10^2	14.0
10^{-6}	10^3	26.9	10^3	13.9
10^{-6}	10^4	24.3	10^4	11.3
10^{-6}	10^5	19.4	10^5	6.4

TABLE V

MINIMUM ATMOSPHERIC NOISE LOSS TO PROPAGATION REGION

Sea Water		Earth	
f	db loss	f	db loss
10^2	416	10^2	79.5
10^3	1170	10^3	149
10^4	3500	10^4	375
10^5	11,000	10^5	1120

a vertical antenna in the propagation region, since all noise components in the propagation region are horizontal. The effect of electrical inhomogeneities, departures from a plane interface, or non-normal incidence of the wave on the interface will be to cause noise components which will be received by the underground antenna. An estimate of the maximum noise power transmitted into the propagation region can be made on the basis that all noise transmitted into the region (considering plane waves at normal incidence on the interface) will be received as disturbing noise. Even this loss is considerable, since a large amount of power is reflected back up to the earth's surface. Table IV shows minimum noise losses to be expected. It is felt that actual losses will be much greater than these.

Total minimum atmospheric noise losses between the above-ground and underground propagation regions, assuming an ocean depth of 1 km or an earth depth of 2 km, are shown in Table V. It can be seen that both the earth and sea water act as very effective shields in preventing atmospheric noise from reaching the region of underground propagation. Almost nothing is known about noise disturbances originating in the earth region of interest (noise generated above or below the region of low conductivity will be attenuated greatly before reaching the region). Tides in the ionosphere under solar and lunar influences cause ionosphere currents which in turn induce earth currents extending perhaps to hundreds of kilometers of depth. Earth currents of longer periods are also produced by magnetic storms associated with solar activity as revealed by sunspots [25]. These ionosphere and earth currents produce electromagnetic disturbances at the earth's surface as evidenced by changes in the earth's magnetic field. These disturbances contain measured frequencies of the order of 1 cps and less [26], [27] and thus present no noise problem for underground communication.

No mechanism is known which would generate underground electromagnetic noise except the thermal noise given by kTB (which also exists above ground and is quite small). Increased temperatures in the vicinity of an underground antenna might raise this value by 1 or 2 db above its value above-ground. Deep in the earth the temperature may reach 5000°K at depths of 2000 km [6], but the kTB value changes only by 12 db, and even this noise could not reach the underground antenna because of absorptive attenuation in the highly conducting region at the greater depths. It is worth noting, though far from conclusive, that noise from undersea sources has not been a problem on the transatlantic telephone cable system [28], [29].

The absence of underground noise sources in the frequency range of interest would make receiver noise a dominant factor in the reception of small signals, a much more attractive limit than that imposed by atmospheric noise. An interesting speculative problem is the following: We assume an above-ground system and an underground system, both operating at the same frequency (100 kc) and compare their performances. Assume equal radiated powers from a vertical antenna in each system. (The analysis then does not consider the relative difficulties of constructing equally efficient antennas.) At a distance of 1000 km over sea water the above-ground signal is virtually unattenuated except for the inverse-square-distance loss. The below-ground signal has the same loss and in addition an absorption attenuation of 67 db (assuming 10^{-7} conductivity). The above-ground receiver, however, may operate in a noise environment of 130 db (see Fig. 4), while the noise of the underground receiver may be only 10 db. Then, for the same radiated power, the receiver of the underground system has the higher signal-to-noise ratio. In fact, the underground system has the same signal-to-noise ratio at about 1700 km that the above-ground system has at 1000 km.

Conclusions

The earth's conductivity below the highly-conducting surface layers is still not known with a high degree of certainty. If it is as low as is indicated by some geophysical data, underground radio wave propagation at VLF will be not merely feasible, but extremely attractive. Lack of underground noise and the effective shielding against atmospheric noise provided by the earth's upper layers make an underground system superior to an above-ground system for long-range communication. In particular, communication with deeply submerged submarines has become possible.

References

[1] H. A. Wheeler, "Radio-wave propagation in the earth's crust," *J. Res. NBS*, vol. 65D, pp. 189–191; March, 1961.
[2] H. Jeffreys, "Earth," in "Encyclopedia Britannica," Encyclopedia Britannica, Inc., Chicago, Ill., vol. 7, pp. 830–837; 1957.
[3] C. L. Drake, *et al.*, "Continental margins and geosynclines: The East Coast of North America north of Cape Hatteras," in "Physics and Chemistry of the Earth," Pergamon Press, New York, N. Y., vol. 3, pp. 110–198; 1959.
[4] W. D. Owsley, "Drilling, petroleum," in "Encyclopedia Britannica, Encyclopedia Britannica, Inc., Chicago, Ill. vol. 7, pp. 662–663; 1957.
[5] H. Hughes, "The Electrical Conductivity of the Earth's Interior," Ph.D. dissertation, University of Cambridge, Cambridge, England; 1953.
[6] G. J. F. MacDonald, "Calculations on the thermal history of the earth," *J. Geophys. Res.*, vol. 64, pp. 1967–2000; November, 1959.
[7] G. D. Garland and T. F. Webster, "Studies of natural electric and magnetic fields," *J. Res. NBS*, vol. 64D, pp. 405–408; July, 1960.
[8] ——, "Studies of natural electric and magnetic fields," *Proc. 1960 Conf. on the Propagation of E.L.F. Radio Waves*, Boulder, Colo., January, 1960, NBS; 1960.
[9] L. Cagniard, "Basic theory of the magneto-telluric method of geophysical prospecting," *Geophys.*, vol. XVIII, pp. 605–635; 1953.
[10] J. R. Wait, "Theory of magneto-telluric fields," *J. Res. NBS*, vol. 66D, pp. 509–541; September, 1962.
[11] T. Cantwell and T. R. Madden, "Preliminary report on crustal magneto-telluric measurements," *J. Geophys. Res.*, vol. 65, pp. 4202–4205; December, 1960.
[12] B. N. Lahiri and A. T. Price, "Electromagnetic induction in non-uniform conductors and the determination of the conductivity of the earth from terrestrial magnetic variations," *Phil. Trans. Roy. Soc. (London), A*, vol. 237, pp. 509–540; January, 1939.
[13] L. M. Brekhovskikh, "Waves in Layered Media," Academic Press, New York, N. Y.; 1960.
[14] K. A. Norton, "The propagation of radio waves over the surface of the earth and in the upper atmosphere," Proc. IRE, vol. 24, pp. 1367–1387; October, 1936.
[15] K. G. Budden, "On the propagation of a radio atmospheric," *Phil. Mag.*, vol. 43, pp. 1179–1200; November, 1952.
[16] ——, "The 'waveguide mode' theory of the propagation of very-low-frequency radio waves," Proc. IRE, vol. 45, pp. 772–774; June, 1957.
[17] J. R. Wait, "The mode theory of VLF ionospheric propagation for finite ground conductivity," Proc. IRE, vol. 45, pp. 760–767; June, 1957.
[18] ——, "Introduction to the theory of VLF propagation," Proc. IRE, vol. 50, pp. 1624–1647; July, 1962.
[19] K. Henney, "Radio Engineering Handbook," McGraw-Hill Book Company, Inc., New York, N. Y., pp. 15–19; 1959.
[20] S. Goldman, "Frequency Analysis, Modulation and Noise," McGraw-Hill Book Company, Inc., New York, N. Y., pp. 213, 254, 401; 1948.
[21] "Revision of Atmospheric Radio Noise Data," International Radio Consultative Committee, Geneva, Rept. No. 65a, 1957.
[22] A. D. Watt and E. L. Maxwell, "Measured statistical characteristics of VLF atmospheric radio noise," Proc. IRE, vol. 45, pp. 55–62; January, 1957.
[23] ——, "Characteristics of atmospheric noise from 1 to 100 kc," Proc. IRE, vol. 45, pp. 787–794; June, 1957.
[24] E. C. Jordan, "Electromagnetic Waves and Radiating Systems," Prentice-Hall, Inc., New York, N. Y.; 1950.
[25] A. G. McNish, "Terrestrial magnetism," "Encyclopedia Britannica," Encyclopedia Britannica, Inc., Chicago, Ill., vol. 21, p. 959; 1957.
[26] W. H. Campbell, "Natural electromagnetic energy below the ELF range," *J. Res. NBS*, vol. 64D, pp. 409–411; July, 1960.
[27] C. W. Horton and A. A. J. Hoffman, "Magnetotelluric fields in the frequency range 0.03 to 7 cycles per kilosecond: Part I, power spectra, and Part II, geophysical interpretation," *J. Res. NBS*, vol. 66D, pp. 487–497; July, 1962.
[28] E. T. Mottram, *et al.*, "Transatlantic telephone cable system—planning and over-all performance," Trans. AIEE, vol. 75 (*Commun. and Electronics*, pt. I), pp. 842–850; January, 1957.
[29] H. A. Lewis, *et al.*, "System design for the North Atlantic link," Trans. AIEE, vol. 75 (*Commun. and Electronics*, pt. I), pp. 850–864; January, 1957.

Correction

J. R. Wait and L. C. Walters, authors of "Curves for Ground Wave Propagation Over Mixed Land and Sea Paths," which appeared on pages 38–45 of the January, 1963, issue of these Transactions, have called the following to the attention of the *Editor*.

Following (7) on page 39, x should be defined by

$$x = \left(\frac{ka}{2}\right)^{1/3} \frac{d}{a}$$

rather than as shown.